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Q-ball inflation

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We show that inflation can occur in the core of a Q ball.

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I. INTRODUCTION

In spite of the great success of quantum field theory, there is still no consistent scenario in which quantum gravity is included. The most promising scenario in this direction would be string theory, where the consistency is ensured by the requirement of additional dimensions and supersymmetry. Supersymmetry is sometimes used to explain the large hierarchy between the scales of grand unified theory (GUT) and the standard model (SM). Unlike the standard model, the supersymmetric standard model (SSM) contains many flat directions in its scalar sector. The flat directions might appear from the moduli fields that parametrize the compactified space of string theory. Moreover, considering the brane extensions of string theory, a flat potential might appear from the potential for the distances between branes, or simply for the positions of branes in the compactified space.[1] In string theory, initially the sizes of extra dimensions had been assumed to be as small as the Planck mass, but later it has been observed that there is no reason to believe such a tiny compactification radius [3]. The idea of the large extra dimension may solve the hierarchy problem. Denoting the volume of the n-dimensional compact space by V_n, the observed Planck mass is obtained by the relation $M_p^2 = M_*^{n+2} V_n$, where M_* denotes the fundamental scale of gravity. If one assumes more than two extra dimensions, M_* may be close to the TeV scale without conflicting any observable bounds. In this scenario the standard model fields are expected to be localized on a wall-like structure and the graviton propagates in the bulk. The most natural embedding of this picture in the string theory context is realized by a brane construction.[2]

On the other hand, we know historically that the characteristic features of the phenomenological models are revealed by discussing their cosmological evolutions. For example, the so-called Polonyi problem for the flat directions of the supersymmetric models has been discussed by many authors [10]. If the historical knowledge still applies to the above brane-extended flat directions, it should be important to seek other options, since the alternative approaches might reveal the underlying problems of the model or show us the novel approach to a new scenario of cosmology.

From the above point of view, we will consider Q balls and show that inflation may start inside a Q ball. It is well known that a variety of models possess flat directions that may lead to nontopological solitons of the Q ball type [11,2]. On the other hand, it is sometimes discussed that topological defects other than the Q balls, which might be produced in the early stage of the Universe, would play important roles in particle cosmology.[3] Here we briefly summarize the interesting differences between conventional defect inflation and the Q-ball inflation.

In conventional models of defect inflation, the typical parameters such as the mass scales or the width of the defect are the constants that do not change with time. In this case, the conditions for defect inflation to start are determined solely by the form of the potential that induces defect configuration. On the other hand, the typical parameters of the Q balls will be determined by their charges that may evolve with time by absorbing other Q balls.

Q balls will be produced during the oscillation after inflation. If the Q balls decay safely, the reheating may be dominated by the "surface reheating" that was discussed in Ref. [15]. On the other hand, if a produced Q ball triggers inflation in its core, the second stage of inflation will start in the local area. Of course the second inflation might produce another Q ball again, which might induce problematic eternal inflation.

The Q-ball inflation might start as the secondary weak inflation that induces the temporal small expansion of a local area. Thus, unlike conventional defect inflation, the Q-ball inflation might be used to explain the bubble structure of the Universe.

In Sec. II we show a simple example of the Q ball that appears on a hybrid potential. Although there had been no explicit argument on such an extension of the flat potential of the Q balls, it is easy to see that the extension of this type is quite natural in many phenomenological models. In Sec. III we examine the conditions for the Q-ball inflation. In Sec. IV, we apply the results obtained in Sec. III to a specific

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[1]Some cosmological implications and defect configurations of the flat directions are discussed in Refs. [1,2].

[2]Constructing successful models for inflation with a low fundamental scale is still an interesting problem [4,5]. Baryogenesis and inflation in models with a low fundamental scale are discussed in [6]. Affleck-Dine baryogenesis [7] in such scenarios is discussed in Refs. [8,9]. We think constructing models of particle cosmology with large extra dimensions is very important since we are expecting that future cosmological observations would determine the fundamental scale of the underlying theory.

[3]Inflation from topological defect is discussed in Ref. [12] for conventional gravity and in Ref. [13] for models of the brane world. Constraints on hidden-sector walls from weak inflation are discussed in Ref. [14].

model of surface reheating [15]. We stress here that the ideas we will show in this paper are quite simple and generic.

II. *Q* BALLS FROM HYBRID POTENTIAL

Here we consider a typical hybrid potential that has been used in models of D-term inflation [16]. As usual, the trigger field ϕ is assumed to have a steep potential, while the field σ parametrizes a flat potential. The explicit form of the effective potential is

$$V(\sigma)=M_0^4\log\left(1+K_1\frac{|\sigma|^2}{M_1^2}\right)+m_{3/2}^2|\sigma|^2\left[1+K_2\log\left(\frac{|\sigma|^2}{M_2^2}\right)\right] \tag{2.1}$$

where $m_{3/2}$ is the gravitino mass, which is obtained by using the supersymmetry breaking scale Λ_{SUSY}, as $m_{3/2}=\Lambda_{\mathrm{SUSY}}^2/M_p$. The constants K_1,K_2 represent the renormalization factors at one-loop, and M_1,M_2 are the renormalization scales. We introduce a dimensionless constant $\eta\equiv M_0/\Lambda_{\mathrm{SUSY}}$ for later convenience. The second term dominates when $\sigma>\sigma_c$, where σ_c is defined as

$$\begin{aligned}\sigma_c&\equiv\frac{M_0^2}{m_{3/2}}\\&=M_p\left(\frac{M_0}{\Lambda_{\mathrm{SUSY}}}\right)^2\\&=M_p\times\eta^2,\end{aligned} \tag{2.2}$$

where the numerical factor is neglected. For $\eta\gg1$, σ_c becomes much larger than the Planck scale ($\sigma_c\gg M_p$), and the second term in Eq. (2.1) is always negligible. On the other hand, for $\eta\ll1$, one should consider two types of *Q* balls [17].

Let us first consider the case where $\eta<1$. As far as $\sigma<\sigma_c$, the potential is dominated by the first term in Eq. (2.1) and the *Q* balls will have the following well-known properties [17,18]:

$$\begin{aligned}&r_Q\simeq\frac{Q^{1/4}}{M_0},\quad\omega\simeq\frac{M_0}{Q^{1/4}},\\&\sigma\simeq M_0Q^{1/4}.\end{aligned} \tag{2.3}$$

On the other hand, when $\sigma>\sigma_c$, the second term will dominate and the *Q* balls will have the following properties [17],

$$\begin{aligned}&r_Q\simeq\frac{1}{\sqrt{|K_2|}m_{3/2}},\quad\omega\simeq m_{3/2},\\&\sigma\simeq|K|^{3/4}m_{3/2}Q^{1/2}.\end{aligned} \tag{2.4}$$

As these results are well established, we use Eqs. (2.3) and (2.4) hereafter to examine the conditions for the *Q*-ball inflation. Because we are considering hybrid potential, we may assume $M_0\gg\Lambda_{\mathrm{SUSY}}$ as well as $M_0\ll\Lambda_{\mathrm{SUSY}}$. If one assumes $M_0\gg\Lambda_{\mathrm{SUSY}}$, η becomes much larger than $O(1)$. In this case, σ is always much smaller than σ_c, which means that one may safely assume that Eq. (2.3) is always satisfied for any Q.

In the next section we use the above results and examine the conditions for the *Q*-ball inflation. We then analyze the properties of the *Q*-ball inflation.

III. INFLATION FROM A *Q* BALL

As in the conventional models of topological inflation [12], the following conditions will be required.

The radius of the *Q* ball must become larger than the Hubble radius. Since the field σ is trapped at a false vacuum within the *Q* ball, inflation will start when the boundary of the *Q* ball exits the horizon.

If we consider the scenario where the charges of the *Q* balls evolve with time, the expectation value of the σ in the core will also change with time. In this case we should examine when the transition from $\sigma<\sigma_c$ to $\sigma>\sigma_c$ occurs. The properties of the *Q* ball will be modified at σ_c.[4]

Naively, *Q* balls might become a black hole before it induces inflation. This condition is rather trivial as we will show later.

Let us examine the above conditions in more detail. First we assume $\eta\ll1$, where both phases $\sigma<\sigma_c$ and $\sigma>\sigma_c$ might appear. When the charge Q is small, the first term in Eq. (2.1) dominates and then Eq. (2.3) is reliable.[5] Since the vacuum energy ρ inside *Q* balls is as large as M_0^4, the Hubble constant when the boundary exits the horizon will be $H\simeq M_0^2/M_p$. Then the condition $r_Q>H^{-1}$ is represented by

$$Q^{1/4}\geq\frac{M_p}{M_0}. \tag{3.1}$$

The above result can be applied as far as $\sigma<\sigma_c$. However, the condition for $\sigma<\sigma_c$ is

$$\begin{aligned}Q^{1/4}&<\left(\frac{M_0}{m_{3/2}}\right)\\&=\left(\frac{M_p}{\Lambda_{\mathrm{SUSY}}}\right)\left(\frac{M_0}{\Lambda_{\mathrm{SUSY}}}\right)\\&=\frac{M_p}{M_0}\eta^2,\end{aligned} \tag{3.2}$$

which is much smaller than Eq. (3.1). Thus in this case, we must conclude that evolving *Q* balls will alter their properties from Eq. (2.3) to Eq. (2.4) before they induce inflation. Of course the evolving *Q* balls must not decay into black holes before they induces inflation. Denoting the Schwarzschild radius by r_g, one can easily find the condition

[4]Of course one might consider the scenario where a huge *Q* ball is produced at the earliest stage of the Universe and inflation starts abruptly.

[5]We are not compelling the scenario where small *Q* balls evolve with time. *Q* balls might be huge when they are produced before inflation.

$$Q^{1/4} \leqslant \frac{M_p}{M_0}, \tag{3.3}$$

which suggests that the critical charge for the black hole formation is much larger than the criteria (3.2).[6]

Now the above results are suggesting that we should consider the Q balls of $\sigma > \sigma_c$ in the case when $\eta \ll 1$. When the Q ball becomes large and the expectation value of the field σ inside the Q ball becomes larger than σ_c, the above criteria are represented by the following conditions. Here the vacuum energy density inside Q balls is estimated by $\rho \simeq m_{3/2}^2 [(|K|^{3/4}) m_{3/2} Q^{1/2}]^2$. The radius of the Q ball will exit the horizon when $r_Q > H^{-1}$, which happens when

$$Q > |K_2|^{-1/2} \left(\frac{M_p}{m_{3/2}}\right)^2 = |K_2|^{-1/2} \left(\frac{M_p}{\Lambda_{\rm SUSY}}\right)^4$$
$$= |K_2|^{-1/2} \left(\frac{M_p}{M_0}\right)^4 \times \eta^4. \tag{3.4}$$

To verify our argument, we should examine if $\sigma > \sigma_c$ is satisfied when inflation starts. $\sigma > \sigma_c$ is satisfied if

$$Q > |K_2|^{-3/2} \left(\frac{M_p}{m_{3/2}}\right)^2 \times \eta^4$$
$$= |K_2|^{-3/2} \left(\frac{M_p}{M_0}\right)^4 \times \eta^8. \tag{3.5}$$

Because we are considering the case $\eta \ll 1$, the condition (3.5) is safely satisfied when inflation starts.

Now we will examine the case $\eta \gg 1$. As we are considering hybrid potential in this paper, $\eta \gg 1$ is not unnatural. As we have discussed above, we should always use Eq. (2.3), because in this case σ_c is much larger than M_p. The conditions for $r_Q > H^{-1}$ and $r_Q > r_g$ are the same as the result obtained above. The properties of the Q balls do not alter as the charge evolves, which is the only difference from the above result. In this case, $\sigma \ll \sigma_c$ is always satisfied for any realistic value of Q. Thus our conclusion for $\eta \gg 1$ is that the Q-ball inflation will start when $Q \geqslant (M_p/M_0)^4$.

In the above discussions we have examined the conditions for inflation to start within Q balls. Our second task is to examine the evolution of the field σ during the Q-ball inflation. At the earliest stage of the Q-ball inflation, the field σ is trapped at the false vacuum because of the large ω. When inflation starts inside the Q ball, the friction term dominates the equation of motion. Then $\dot{\sigma}$ decays as $\dot{\sigma} \sim e^{Ht}$. Assuming that the change in $|\sigma|$ is much slower than that in ω, one can obtain

$$\omega \simeq \omega_0 e^{Ht}, \tag{3.6}$$

where ω_0 denotes the initial value of ω when inflation starts. In this case, one may expect two kinds of inflation that may start subsequently. The first inflation occurs during the period of $\omega^2 > V''$ when the field σ is trapped at the false vacuum because of the large ω. The e-foldings of the first inflation is

$$N_e \simeq \frac{1}{2} \log\left(\frac{w_0^2}{V''}\right), \tag{3.7}$$

where V'' is the effective mass of the field σ at the false vacuum. Then after this period, the conventional slow-roll inflation will start if some conditions are satisfied. If $\eta > 1$, the Hubble parameter might be smaller than the effective mass, where the situation is the same as the conventional D-term inflation. The e-foldings of the second inflation is determined by the charge of the Q ball, which determines the initial value of σ. Even if the slow-roll inflation does not start, the expansion of the local area induced by the small inflation will affect the later cosmological structure formation. In any case, the cosmological observations of the present Universe might put some bound on the Q-ball inflation, which might put a bound on the phenomenological models.

Our last example is the Randall-Sundrum type 2 (RS-2) model [19]. In this case the Friedman equation receives an additional term that is quadratic in the density. The Hubble parameter is related to the energy density by

$$H^2 = \frac{8\pi}{3M_p^2} \rho \left(1 + \frac{\rho}{2\lambda}\right), \tag{3.8}$$

where λ is the brane tension. Denoting the ratio between ρ and λ by $\epsilon \equiv \lambda/\rho$, the modified condition for $H^{-1} < r_Q$ is relaxed when $\epsilon < 1$. Assuming that $\epsilon \ll 1$, one can obtain for $\sigma < \sigma_c$,

$$Q \geqslant \left(\frac{M_p}{M_0}\right)^4 \times \epsilon^2. \tag{3.9}$$

For $\sigma > \sigma_c$, it becomes

$$Q > |K_2|^{-1/2} \left(\frac{M_p}{M_0}\right)^4 \times \eta^4 \epsilon. \tag{3.10}$$

From Eqs. (3.9) and (3.10), one can see that the condition for Q-ball inflation is rather relaxed in the Randall-Sundrum type 2 scenario.

IV. SURFACE REHEATING AND Q-BALL INFLATION

In this section we will examine whether the Q balls produced just after inflation lead to the surface reheating or to the Q-ball inflation. The surface reheating in models of run-

[6]To understand Eq. (3.3), we think it is helpful to consider a simplest toy model. Imagine a spherical region with the radius r_b. The vacuum energy is $\rho = \rho_b > 0$ inside the "ball," while $\rho = 0$ outside. Then the mass of the "ball" is given by the formula $M_b = (4\pi/3) r_b^3 \rho_b$. The Schwarzschild radius of the "ball" is $r_g^2 \simeq \rho_b / M_p^2$. On the other hand, the Hubble parameter inside the "ball" when the boundary exits the horizon is $H^2 \simeq \rho_b / M_p^2$. Thus the naive black hole condition will give a rather trivial result.

ning mass inflation is already examined in Ref. [15]. Here we mainly follow the setups of [15], and discuss if the *Q*-ball inflation takes place. The potential which can lead to a *Q*-ball formation is

$$V(\sigma)=m^2\left(1+|K|\log\left[\frac{\sigma^2}{M_p^2}\right]\right). \tag{4.1}$$

We will assume that the maximum charge of a *Q* ball might be as large as the maximum charge within the Hubble radius when *Q* balls are produced. Denoting the Hubble parameter and the expectation value of the field σ at the time of the *Q*-ball formation by H_Q and σ_Q, one can obtain the maximum charge $Q_{\rm MAX}$,

$$Q_{\rm MAX}\simeq\frac{\omega_Q\sigma_Q^2}{H_Q^3}. \tag{4.2}$$

In generic situations, ω_Q is expected to be about the same order as the Hubble parameter H_Q. Finally, one can obtain the simple result,

$$Q_{\rm MAX}\simeq\sqrt{K}\left(\frac{M_p}{m}\right)^2, \tag{4.3}$$

which is nearly the same order as Eq. (3.4).[7] However, it becomes much smaller than Eq. (3.4) if $K\ll 1$. Although it is quite difficult to calculate the exact values of the properties of the cosmological *Q* balls, it seems fair to conclude from the above rough estimations that the surface reheating will be reliable if *K* is much smaller than $O(1)$.[8]

V. CONCLUSIONS AND DISCUSSION

In this paper we have examined the idea that a modified version of the conventional topological inflation might start within *Q* balls. An extension of the *Q* balls is discussed by using an explicit hybrid potential, which is useful for our discussion. As the flat direction might appear in the hybrid potential of the brane distance, *Q* balls might appear for the brane rotation [2]. We have obtained the criteria for the *Q*-ball inflation, which is comparable to the phenomenological values of the conventional *Q*-ball formation. Since the *Q*-ball formation is quite general in many models where flat directions are contained in the low-energy effective Lagrangian, we think the cosmological bound that will be obtained by considering *Q*-ball inflation is quite important.

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[7]Note that *m* in Eq. (4.2) corresponds to $m_{3/2}$ in Eq. (3.4).

[8]This result does not exclude the possibility that a few *Q* balls might become abnormally large absorbing other *Q* balls.

[1] T. Matsuda, ''Yukawa cosmology and intersecting brane models,'' hep-ph/0309314.

[2] T. Matsuda, J. High Energy Phys. **09**, 064 (2003).

[3] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G.R. Dvali, Phys. Lett. B **436**, 257 (1998); I. Antoniadis, *ibid.* **246**, 377 (1990); N. Arkani-Hamed, S. Dimopoulos, and G.R. Dvali, *ibid.* **429**, 263 (1998).

[4] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and J. March-Russell, Nucl. Phys. **B567**, 189 (2000); R.N. Mohapatra, A. Perez-Lorenzana, and C.A. de S. Pires, Phys. Rev. D **62**, 105030 (2000); A.M. Green and A. Mazumdar, *ibid.* **65**, 105022 (2002); D.H. Lyth, Phys. Lett. B **448**, 191 (1999); **466**, 85 (1999); T. Matsuda, Phys. Rev. D **66**, 107301 (2002).

[5] T. Matsuda, Phys. Rev. D **67**, 083519 (2003); J. Cosmol. Astropart. Phys. **06**, 007 (2003); ''F-term, D-term and hybrid brane inflation,'' hep-ph/0302078.

[6] G.R. Dvali and G. Gabadadze, Phys. Lett. B **460**, 47 (1999); T. Matsuda, Phys. Rev. D **66**, 023508 (2002); **65**, 107302 (2002); **64**, 083512 (2001); **66**, 047301 (2002); J. Phys. G **27**, L103 (2001); A. Masiero, M. Peloso, L. Sorbo, and R. Tabbash, Phys. Rev. D **62**, 063515 (2000); A. Pilaftsis, *ibid.* **60**, 105023 (1999); R. Allahverdi, K. Enqvist, A. Mazumdar, and A. Perez-Lorenzana, Nucl. Phys. **B618**, 377 (2001); S. Davidson, M. Losada, and A. Riotto, Phys. Rev. Lett. **84**, 4284 (2000).

[7] I. Affleck and M. Dine, Nucl. Phys. **B249**, 361 (1985).

[8] A. Mazumdar, Nucl. Phys. **B597**, 561 (2001); Phys. Rev. D **64**, 027304 (2001); A. Mazumdar and A. Perez-Lorenzana, *ibid.* **65**, 107301 (2002); R. Allahverdi, K. Enqvist, A. Mazumdar, and A. Perez-Lorenzana, Nucl. Phys. **B618**, 277 (2001); T. Matsuda, Phys. Rev. D **65**, 103502 (2002).

[9] T. Matsuda, Phys. Rev. D **65**, 103501 (2002); **67**, 127302 (2003).

[10] G.D. Coughlan, W. Fischler, E.W. Kolb, S. Raby, and G.G. Ross, Phys. Lett. **131B**, 59 (1983); T. Banks, D.B. Kaplan, and Ann E. Nelson, Phys. Rev. D **49**, 779 (1994); S. Weinberg, Phys. Rev. Lett. **48**, 1303 (1982).

[11] S.R. Coleman, Nucl. Phys. **B262**, 263 (1985); A. Kusenko, Phys. Lett. B **405**, 108 (1997).

[12] A. Vilenkin, Phys. Rev. Lett. **72**, 3137 (1994); A.D. Linde, Phys. Lett. B **327**, 208 (1994).

[13] Matsuda, the second citation of Ref. [5].

[14] T. Matsuda, Phys. Lett. B **486**, 300 (2000).

[15] K. Enqvist, S. Kasuya, and A. Mazumdar, Phys. Rev. Lett. **89**, 091301 (2002).

[16] P. Binetruy and G.R. Dvali, Phys. Lett. B **388**, 241 (1996); E. Halyo, *ibid.* **387**, 43 (1996); T. Matsuda, *ibid.* **423**, 35 (1998).

[17] S. Kasuya and M. Kawasaki, Phys. Rev. Lett. **85**, 2677 (2000).

[18] G.R. Dvali, A. Kusenko, and M.E. Shaposhnikov, Phys. Lett. B **417**, 99 (1998).

[19] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).